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ABSTRACT

This research investigated developmental shifts in the character of children's tool using activity in the domain of scale drawing. Fifty-five children from three grade levels (grades 3, 5, and 7) were individually interviewed as they participated in both enlarging and reducing a one-dimensional object, the letter "F," to a scale of four. Children participated in the enlarging/reducing activity under different tool conditions: once using a ruler, once using graph paper, and once using a moveable replica of the letter F. Three principal features of children's activities were analyzed: (1) the accuracy of children's constructions; (2) the ways children incorporated the tool into their problem solving; and (3) children's scale-linked understandings. Statistical analysis revealed that children's accuracy, their tool-using activity, and their scale understandings shifted with age, and these shifts interacted with the difficulty of the scale task (enlarging vs. reducing the F) and tool condition (replica vs. graph paper vs. ruler). Results support L. Vygotsky's work on tool use illustrating the development of sign operations, but his analysis is extended to incorporate children's conceptual understandings in the development process. An appendix contains three tables of means and standard deviations of study variables. (Contains 5 tables, 10 figures, and 53 references.) (Author/SLD)

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Children's Problem Solving in Scale Drawing:

How are Conceptual Understandings and Tool Use Interwoven?

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Paper presented at the 1998 annual meeting of the American Educational Research Association, San Diego, CA.

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Running Head: Tool use in scale drawing

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(This work was part of the author's dissertation research conducted while at the University of California, Los Angeles)

Abstract

This research investigated developmental shifts in the character of children's tool using activity in the domain of scale drawing. Fifty five children from three grade levels (grades 3, 5, and 7) were individually interviewed as they participated in both enlarging and reducing a one-dimensional object, the letter "F", to a scale of four. Children participated in the enlarging/reducing activity under three different tool conditions; once using a ruler, once using graph paper, and once using a moveable replica of the letter F. Three principal features of children's activities were the focus of analysis: the accuracy of children's constructions, the ways children incorporated the tool into their problem solving, and children's scale-linked understandings. Statistical analyses revealed that children's accuracy, their tool using activity and their scale understandings shifted with age, and these shifts interacted with the difficulty of the scale task (enlarging vs. reducing the F) and tool condition (replica vs. graph paper vs. ruler). Results support Vygotsky's work on tool use illustrating the development of sign operations, but his analysis is extended to incorporate children's conceptual understandings in the development process.

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Introduction

This research is part of a larger study (Sloan, 1996) that investigated developmental shifts in children's use of manipulatives ("tools") in mathematical problem solving. There are both conceptual and applied reasons for studying tool use. From a conceptual standpoint, tools are of central concern in Vygotsky's (1978; 1986) treatment of cognitive development and in recent sociocultural accounts of cognitive development (see e.g., Cobb, April, 1993; Cole & Cole, 1989; Greenfield, 1995; Lave, 1988; Pea, 1993; Rogoff, 1990; Rogoff & Lave, 1984; Saxe, 1991; Wertsch, 1991). A focus of these treatments is on developmental shifts in the character of children's tool using activities and how the appropriation of a tool leads to a reorganization of the child's problem solving activity. Rarely do researchers offer systematic, empirical analyses of developmental shifts in children's tool using activity or the way variation in tools leads to differentiation in the character of children's problem solving within a particular subject matter domain.

From an applied standpoint, children's use of tools is central to recent mathematics education reform ideas (see e.g., California State Board of Education, 1992; National Council of Teachers of Mathematics, 1991). Empirical contributions to the analysis of tool use can too inform current approaches to classroom based assessment and instruction. As a domain for study, I focus on children's construction of scale drawings, specifically enlarging and reducing objects to scale. The domain of scale is used because as a complex measurement domain it provides a context in which tool use is virtually essential. Further, in the context of ongoing educational reforms, measurement is increasingly a curricular focus.

In the sections below, I discuss tool use in Vygotsky's general treatment of cognitive development, extending his treatment to an analysis of scale-linked mathematical understandings implicated in children's problem solving in scale drawing. Examples of children's problem solving in pilot work guides this analysis. I conclude the introduction with a discussion of how three different tools used in the piloted scale drawing problem--a ruler, graph paper, and a moveable replica of the scaling object--can differentially affect problem solving and interact differently with children's developing understandings of scale in their problem solving performances.

Conceptual Formulation of Tool Use and Cognitive Development

As part of his general treatment of cognitive development, Vygotsky (1978, 1986) argued that in infancy individuals' activities are initially controlled by the environment (e.g., direct stimulus-response reactions in infants), but over development eventually become controlled by the individual. He proposed that individuals gain control over their environment by appropriating sociocultural artifacts and supports—sign forms, social interactions, scientific concepts, tools—in order to mediate these interactions with the environment (Saxe, 1991). The use of signs and tools leads humans to a specific structure of behavior that moves away from biological development and creates new forms of a culturally-based psychological process (Vygotsky, 1978).

Sign Operations

One of Vygotsky's (1978) analyses of sign using or tool using activities involves sign form use in the emergence of mediated or voluntary memory. In experiments conducted by A. N. Leontiev, individuals (pre-school children, school-age children, and adults) were first asked to answer a variety of questions, some requiring a color for an answer, others not. For instance questions were "what color is your shirt?", "have you a playmate?", and "what color is a lemon?" (Vygotsky, 1978, p. 41). In a second run through the questions, the individuals were told they could not use the same color name twice and there were two forbidden colors that they were not allowed to use at all. In a third run through the questions, individuals were given nine color cards to use as aids.

The experiment revealed that whereas pre-school children performed better without than with cards, school-aged children performed better with cards, and adults performed equally as well with or without cards. In addition, adults performed better overall than all the children.

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Vygotsky (1978) interpreted these results to indicate that there are three basic stages of mediated remembering. First at the pre-school age the child is incapable of mastering his or her behavior by organizing the external stimuli or tool. The cards do not serve any instrumental function. At the second stage, there is a marked improvement in performance with the introduction of the cards; the external tool predominates--a psychological instrument acting from the outside. At the third stage (among adults) behavior is still mediated but the external tool is emancipated from primary external forms. "What takes place is what we have called internalization; the external sign that school children require has been transformed into an internal sign produced by the adult as a means of remembering" (p.45).

Critique of Tool Use Literature

Vygotsky's analyses of tool use in the mediation of memory offer a framework for understanding general developmental shifts in tool use in problem solving activity. However, his framework does not consider an individual's developing understandings in the problem solving domain. I, like others (e.g., Cobb, April 1993, 1994; Cole & Griffin, 1993; Pea, 1993; Piaget in Ginsburg & Opper, 1988; Saxe, 1991; Smagorinsky, 1995) argue that shifts in tool use cannot be isolated from shifts in understandings in the task domain. This is especially evident in more complex domains such as mathematics, where potentially many higher order understandings are involved and incorporated in children's organization of their tool using activity.

Developmental Shifts in Children's Understandings of Scale

Scale, as it is conceptualized in this research, is "the proportion which the representation of an object bears to the object itself; a system of representing or reproducing objects in a smaller or larger size proportionately in every part" (Oxford English Dictionary, 1989, p. 561). The activity of scaling that is of interest to this research is that which an engineer, architect or topographer might be involved in, namely enlarging and reducing objects to a quantifiable scale.

To date, empirical research in children's understandings of scale is not vast and only a few of its functions have been explored. For instance, Carraher (1986) compared school children's and construction foremen's abilities to find scales of blueprint drawings; DeLoache (1991a, 1991b) studied children's understandings of a scale model; Goldenberg (1988a, 1988b) analyzed children's understandings of scale issues related to graphing mathematical functions; and Millroy (1992), as part of an ethnographic study, describes scaling strategies of South African carpenters. The existing work is not very informative for an analysis of developmental shifts in children's concepts of scale. In fact, to create a preliminary analysis of children's developing understandings of scale drawings, I found more useful empirical work in the scale-linked understandings of proportions/ratios and fractions--concepts that are central to problem solving in scaled enlargements and reductions. In addition, I used pilot work data in which children enlarged and reduced three different objects (a toothpick, the letter "L", a rectangular "battery") with a choice of three different tools (a ruler, graph paper, and a replica of the scaling object) to a scale of 4 (see Sloan, 1996 for discussion of pilot work).

Probably the most integral sub-concept of scale, and that which is central to problem solving in both scaled enlargements and reductions is proportional reasoning and concepts of ratio (Behr, Harel, & Post, 1992; Carraher, 1986; Carpenter, Fennema, & Romberg, 1993; Hart, 1988; Karplus, Pulos, & Stage, 1983; Lamon, 1990, 1993; 1994; Lesh, Post, & Behr, 1988; Noelting, 1980a, 1980b; Tourniaire & Pulos, 1985; Guershon & Confrey, 1994). In Table 1, I present a developmental analysis of scale-linked understandings, developed through analysis of existent research reports and my own pilot observations. To summarize Table 1, children's scale-linked understandings shift from qualitative, to quantitative additive, to multiplicative in nature. These understandings are relevant to both enlargements and reductions.

In drawing scaled reductions, children are involved with a meaning of fractions as a "part of a whole", or as the result of partitioning an object of continuous quantity (linear or area) into parts (for fractions meanings see Behr, Lesh , Post, & Silver, 1983; Kieren, 1975; Nesher, 1985; Ohlsson, 1988). Within the partitioning work, researchers developed schemes of developmental shifts in fractions understandings (see e.g., Piaget, Inhelder, and Szeminska, 1960; Potheir and Sawada, 1983). Using this work and that of others (e.g., Kieren & Nelson, 1978; Kieren & Southwell, 1979; Kieren, Nelson, & Smith, 1983) integrated with pilot work analyses, I developed a scheme of scale reduction understandings that have implications for problem solving in the reductions task domain (see Table 2 for proposed developmental shifts in children's reduction understandings).

Differential Effect of Tools in Developmental Shifts in Tool Use

Aside from individuals' domain-linked understandings, another aspect missing in Vygotsky's tool using work is consideration of the differential effects of variations in tools. I expected the particular tools children used would interact with domain-linked understandings and differentially affect developmental shifts in tool use. For instance, a child with a multiplicative scale understanding can use a ruler to enlarge by either measuring the original and multiplying the number of units by the scale factor, or by marking the original length on the ruler and moving the ruler iteratively four times. In the latter case, the child does not take advantage of the equal interval, value laden units of the ruler and tool using activity is very different from the prior example although underlying domain-linked understandings are similar. Pilot study observations and empirical work on the effects of tool variation on problem solving (see e.g., Guberman, 1992; Saxe & Moylan, 1982) aided in the formulation of this expectation. Thus I analyzed children's scaling efforts to enlarge and reduce as they deployed three different tools—a ruler, graph paper, and a replica of the scaling object.

Methods

Subjects

Subjects were 55 students; 19 third graders (9 boys, 10 girls), 18 fifth graders (8 boys, 10 girls), and 18 seventh graders (10 boys, 8 girls) from 3 different schools (one grade from each school) in middle income areas of Santa Barbara, California. Approximately 65% of the children were Caucasian, 20% of Latino descent, and the final 15% comprised of African Americans, Asian Americans and others.

Procedure

Scale Relations Interview

Students were individually interviewed about problem solving strategies as they carried out scale drawing tasks. Subjects were presented with a story in which they needed to help a painter draw the letter "F" for the Franco family's front door and another "F" for the Franco family's mailbox. Each subject enlarged and reduced the letter F under the three different tool conditions (note: the story line in each condition differed slightly and the procedure was counterbalanced across subjects, within grade to control for order effects). Thus each subject drew six F's. Interviews were video taped and lasted approximately 20-40 minutes.

Measurement Skills Pretest and Training

Several days prior to participating in a scale drawing interview, students were administered a written test on measurements skills with three different tools; a ruler, graph paper, and a paper clip. Just prior to the interview, a 15-30 minute individual training was provided for subjects who erred on more than one out of 6 possible problems on a particular tool. The purpose of the pretest and training was to control for "experience" with the tools.

Coding

Children's interviews were coded from the videotaped records. Three kinds of coding were accomplished: Accuracy (the absolute difference in inches between the child's drawing and the correct drawing), Tool Use (based on Vygotsky's analyses, see Table 3), and Scale Enlargement and Scale Reduction Understandings (based on analyses of pilot work and prior empirical work, see Tables 4 and 5).

Although Tool Level codes and Understanding Level codes are based on the same task-linked behaviors, the tool scheme targets how the tool was used to mediate behavior and the understanding schemes target the level of understandings implicated in the strategy. To illustrate the differences in these coding schemes, consider the following strategy to draw an enlarged segment; a child marks the original segment on the graph paper with his fingers and moves his fingers across the paper iteratively, four times. This strategy would receive a Tool Level 3 code--the child is using the tool as an external mediator of problem solving--and an Enlargement Understanding Level 4 code--the child's understanding of scale is quantitative and multiplicative. Had the child counted the length of the original segment in graph paper squares and multiplied the amount by four, this strategy would be assigned the more advanced Tool Level 4 code--the function of the tool to determine the scaled length shifts to mental status--but still the same multiplicative understanding code (Enl Und Level 4).

To determine intercoder reliability, a second coder coded a random sample of 10% of the subjects from each grade and percent agreement for each part of the F was computed. The following are percent agreements between coders for each tool condition (adjusted for chance by Cohens' cappa): Ruler enlargement 70.9%; Graph Paper enlargement 91.7%; Replica enlargement 95.8%; Ruler reduction 62.6%; Graph paper reduction 87.5%; Replica reduction 66.7%.

Results

Creating Competence Measures from Codes

To understand the influence of GRADE, CONDITION, and DIRECTION on Tool Use and Understanding, I created "competence indices" that aggregated students performances across the four line segments on the F. To accomplish this, I computed the mean[1] scores for accuracy, tool use, and scale understanding for each student. These indices were used in GRADE x CONDITION x DIRECTION ANOVAs[2].

In addition, I created percent distributions of children's levels for tool use and scale understanding as a function of GRADE and CONDITION. These distributions are frequency counts of all 4 scores (one for each segment of the F) attained by each subject, and thus preserve the ordinal property of Tool Use and Understanding level.

Analysis on Accuracy of Scaling

Figures 1 and 2 contain distributions of the means for error scores (in inches) for F enlargements and reductions, respectively. Error scores were analyzed using a 3 (GRADE) X 3 (CONDITION) X 2(DIRECTION) split plot ANOVA. The ANOVA revealed main effects for GRADE ($F(2,52)=49.73$, $p<.001$) and DIRECTION[3] ($F(2,52)=58.47$, $p<.001$) and 2-way interaction effects between GRADE and CONDITION ($F(4,104)=3.61$, $p<.01$) and between GRADE and DIRECTION ($F(2,52)=22.52$, $p<.001$). There was also a 3-way interaction effect between GRADE, CONDITION, and DIRECTION ($F(4,104)=3.12$, $p<.05$).

Grade

To determine the source of the main effects and interactions I conducted ONEWAY ANOVAs for GRADE on error scores. Results revealed that for both enlargements and reductions, children's accuracy increased between third and fifth grade, but remained flat between the fifth and seventh grades.

Tool Condition

[1] Because Tool Use and Understanding scores are ordinal data, analyses using the median score across the 4 parts of the F were also run. Results were the same for medians as they were for means, so means are reported here.

[2] Although tool use and understanding scores were ordinal data, the decision to use ANOVAs for analysis was based on the purpose of this analysis; to test hypotheses about the developmental progression of Tool Use and mathematical Understandings, and the variations of these progressions under certain conditions (tool and direction conditions). The "distance" between categories of this progression was not of central concern. Because I wanted to test hypotheses about interaction effects, it was better to use ANOVA's than non-parametric tests, but the "equivalent" non-parametric tests were performed as well (i.e., where one-way ANOVA's were used, either a Kruskal-Wallis one-way ANOVA, or a Mann-Whitney U test was performed). Most results from non-parametric tests confirmed results obtained from ANOVA's; footnotes indicate what non-parametric tests were performed when outcomes differed from parametric tests

[3] The main effect for DIRECTION is not meaningful here because there is a much greater potential for accuracy error with enlargements than with reductions.

ONEWAY ANOVAS for CONDITION showed that for *enlargements*, third and fifth graders did not differ in their accuracy across conditions, whereas seventh graders were more accurate in the Graph Paper and Ruler conditions than in the Replica condition. For *reductions*, third graders were more accurate in the Replica than the other tool conditions, whereas there was no difference in fifth and seventh graders' accuracy across tool conditions.

Analysis on Shifting Forms of Mediation in Tool Using

Figures 3 and 4 contain the means for Tool Use indices as a function of GRADE and CONDITION for enlargements and reductions, respectively. Because the range of possible scores for Tool Use differed between the Replica condition (range = 1-3) and the Ruler/Graph Paper conditions (range=1-4), one 3(GRADE)) X 2(CONDITION) X 2(DIRECTION) split plot ANOVA was conducted that included Ruler and Graph Paper conditions, and one 3(GRADE) X 2(DIRECTION) split plot ANOVA that included the Replica condition was conducted.

ANOVAs for Ruler and Graph Paper conditions revealed main effects for both GRADE ($F(2,52)=46.00$, $p<.001$)) and DIRECTION ($F(1,52)=27.19$, $p<.001$), and a 2-way interaction between GRADE and DIRECTION ($F(2,52)=7.54$, $p<.01$). There was also a 3-way interaction between GRADE, CONDITION, and DIRECTION ($F(2,52)=4.32$, $p<.05$). The ANOVA for the Replica condition revealed main effects for GRADE ($F(2,52)=14.76$, $p<.001$) and DIRECTION ($F(1,52)=27.60$, $p<.001$) but no interaction effects.

Grade

Regarding GRADE related shifts, Duncan post hocs revealed that for both enlargements and reductions, in the Graph Paper and Ruler conditions seventh graders performed better than fifth who performed better than third, however, for the Replica condition, seventh graders did not perform better than fifth graders.

Tool Condition

Because the above analyses could not reveal CONDITION effects between the Replica and the other conditions, to analyze CONDITION effects between all three conditions, Level 4 scores were collapsed to Level 3 so that levels in all three conditions ranged from 1-3; then indices scores were recalculated using the collapsed scores. Duncan post hocs on CONDITION using collapsed scores revealed that for *enlargements*, third graders performed better in the Replica than the Ruler and Graph Paper conditions. For *reductions* there were no significant differences in any grade across conditions.

For *enlargements*, whereas third graders performed at higher Tool Use scores in the Replica than the Graph Paper and Ruler conditions, seventh graders performed better in the Graph Paper and Ruler conditions than in the Replica. Fifth graders performed the same across the three tool conditions.

Direction

Duncan post hocs revealed that subjects performed significantly better in the enlargement than reduction domain, however this varied by tool condition; third graders' Tool Use scores were significantly higher in enlargements in the Replica condition; fifth graders' were higher in the Ruler condition, and seventh graders' were higher in the Ruler and Graph Paper conditions.

Analysis on Developing Understandings of Scale Relations

Figures 5 and 6 are distributions of Understanding indices for enlargements and reductions, respectively. Because Understanding scales differ for enlargement and reduction domains, it was necessary to keep direction constant when analyzing shifts in scale understandings across tool conditions[4]. Hence, I ran two 3(CONDITION) X 3(GRADE) split plot ANOVAs--one for enlargements and one for reductions. For enlargements there was a

[4] See Sloan, 1996 for extended analysis of DIRECTION on Understanding scores

main effect for both GRADE ($\underline{F}(2,52)=45.16$, $\underline{p}<.001$), and CONDITION ($\underline{F}(2,104)=3.30$, $\underline{p}<.05$) and a GRADE X CONDITION interaction ($\underline{F}(4,104)=3.40$, $\underline{p}<.05$). For reductions, there was only a main effect for GRADE ($\underline{F}(2,52)=27.11$, $\underline{p}<.001$).

Grade

Duncan post hocs on GRADE for both enlargements and reductions revealed that for Replica and Ruler conditions, the sophistication of fifth and seventh graders' understandings did not differ significantly, but were more sophisticated than third graders'. In contrast, in the Graph Paper condition seventh graders' understandings were significantly higher than fifth graders'[5].

Tool Condition

Additional post hocs revealed that for *enlargements*, third grader's understandings were more sophisticated in the Replica than in the Graph Paper condition.

Discussion

The purpose of this research was to offer systematic developmental analyses of tool using activity in a mathematical domain (scale drawing) and in so doing, provide evidence for the argument that children's developing domain-linked mathematics understandings affect their organization of tool use and problem solving. Furthermore, the design varied the tools children used in problem solving in order to study these effects on the interplay between tool using activity and children's scale understandings.

Shifting Organization of Tool Using Activities: Effects of Age, Tool Condition, and Conceptual Requirements

Age and Tool Condition: Discussion of GRADE and CONDITION Effects

Analyses on Tool Use indices shed light on sources behind the accuracy of children's scale drawings. For enlargements, ANOVAs revealed that tool use became increasingly more sophisticated between the third and fifth grades, indicating that the increasing accuracy of scale drawings was due to increasing adequacy in tool use and hence problem solving. Percent distributions (see Figures, 1, 8, and 9) further substantiate this interpretation by revealing that third grader's tool use led primarily to inadequate solutions (notice that Levels 1 and 2 dominate), solutions that became more adequate at the fifth and seventh grades.

For the Ruler and Graph Paper conditions, the sophistication of tool use also increased from the fifth to the seventh grades, indicating that while accuracy remains the same, the means by which fifth and seventh graders accomplished scaling differed. For enlargements, percent distributions reveal that although seventh graders' tool using activity involved more mental calculations of scale (Tool Level 4), much of both fifth and seventh graders' tool use led to adequate--hence accurate--solutions. Fifth graders were more likely to use the tool as an external support for determining the scaled length (Tool Level 3). For instance, they might mark the length of the original segment on the ruler and move the ruler iteratively four times. Important to note is that understandings for both strategies then, were also necessarily multiplicative--recall there was no difference in Understanding means (also see percent distributions, figs 10, 11, and 12)--but tool use varied because it was possible to mediate multiplicative problem solving in more or less sophisticated ways with the tool.

For the Replica condition, because the replica lacked the properties (namely equal interval partitions) that allowed for efficient, mental calculation of the scaled length, seventh graders were forced to use less sophisticated strategies--resulting in less accuracy--hence their mean Tool Use equalled that of fifth graders'. In contrast, third graders engaged in more

[5] However, the pattern of fifth and seventh graders' means in the Ruler enlargement condition were very similar to the Graph Paper enlargement, and in fact, results from non-parametric analyses (Kruskal-Wallis ONEWAY ANOVA) indicate that in the Ruler enlargement condition, seventh graders' scores were significantly higher than fifth graders' ($X^2(1,n=36)=6.45$, $P<.05$).

sophisticated tool use with the replica than with the other tools (although accuracy did not reflect this for enlargements[6]).

A notable trend observed in interviews was the contrast in strategies deployed by the third graders in the Replica condition as compared to the other conditions. With the replica, many third graders successfully drew 4 iterative segments for each enlarged segment. While this strategy is indicative of multiplicative reasoning, they often reverted to inaccurate, qualitative-ordinal strategies in the Ruler or Graph Paper conditions (remember students created all 3 drawings at the same sitting and there was often a glaring lack of equivalence between drawings). Under these tool conditions, some third graders did not use the tool at all (Enl. Tool Level 1), or used the tool only as a straight edge (Enl. Tool Level 2). Although the increase in size was arbitrary, the resultant drawing was usually relatively proportionate. At times, however, in what was perhaps an effort to satisfy the problem of scaling "4 times" larger, the child drew each segment of the "F" 4 inches long, or 4 graph paper squares long, resulting in a markedly disproportionate "F". Thus the ruler and graph paper actually hindered their solutions.

Conceptual Requirements: Discussion of DIRECTION effects

Reducing objects to scale require additional partitioning and fractions concepts unnecessary when enlarging, making it a much more difficult problem to solve. In the reductions domain, we saw an overall shift down in Tool Use and Understanding levels[7] (See Percent Distributions in Figures 7-12), the extent to which varied by Grade and Tool Condition.

Third graders' tool use was more sophisticated for the Replica enlargement than for the reduction (there was no difference in DIRECTION for the Ruler or Graph Paper). While often capable of multiplicative strategies when enlarging (Enl. Tool Level 3), they reverted to "estimation" strategies when reducing (Red. Tool Level 1)--a result I attribute to the more difficult conceptual requirements of reducing. Third graders were, however, more *accurate* using the replica for reducing than when using the other tools, although their use of the tool was not more sophisticated. While their estimation strategies with the replica resulted in a relatively proportionate, smaller F, this same strategy with the ruler and graph paper resulted in more disproportionate and larger F's. Again, some tried to incorporate the "4" into their strategy (like the "4 square/inch" strategy above).

Fifth and seventh graders' tool use was more sophisticated for Ruler[8] and Graph Paper enlargements than for reductions. Again, more taxing conceptual requirements in reducing resulted in less sophisticated tool use, in this case with the more "difficult" tools--difficult in that multiplicative operations are less transparent with the ruler and graph paper. Percent distributions (Figure 8) indicate Fifth graders used more lower level strategies to reduce, including additive strategies (Red. Tool Level 2) whereby they would subtract four units--four graph paper squares, for instance-- from the original dimension. This posed an interesting problem because the bottom horizontal line of the F was four graph paper squares long to begin with, so the "subtraction" strategy resulted in zero. Some students explained that the answer was "zero"; their drawing then looked like and upside down "L".

When fifth and seventh graders used the replica there was no significant difference in tool use between reductions and enlargements. Thus, whereas third graders' tool use levels were higher in the Replica Enlargement than Reduction condition, seventh graders' levels were higher with the Ruler and Graph Paper enlargements, and fifth graders' levels were higher with the Graph Paper enlargements.

[6]A lack of effect for accuracy may have been due to third graders' gross inaccuracies as evidenced by standard deviations (Appendix A), or by their decreased manual dexterity abilities; third graders had a much more difficult time lining up endpoints and keeping lines straight as they drew the scaled figure.

[7] Recall differences in accuracy are not meaningful due to greater error potential with enlargements

[8] 7th grade means were significantly higher than 5th's only when using non-parametric analyses: Kruskal-Wallis ONEWAY ANOVA ($X^2(1,n=36)=6.45, P<.05$).

Summary

To summarize effects linked to differentiation in tools, all three grades used more Level 3 Tool Use strategies with the replica than with the other tools--in other words, they tended to mediate multiplicative problem solving where the replica was a necessary, external support. This marked a significant shift down in Tool Use levels for seventh graders, a significant shift up for third graders, while fifth grade scores remained the same. While partitions and values were used to the seventh graders' (and to a lesser extent, fifth graders') advantage in the Graph Paper and Ruler conditions, these properties seemed to hinder the performance of third graders. Their qualitative and "quantitative-graphic" notions of scale proved much more conducive to scaling with a tool such as the replica where multiplication operations are much more transparent. In other words, third graders were capable of counting out four sides with the replica, but became incapable of this somewhere in the process of scaling with a ruler or graph paper; whether it was in translating segments to other units of measure, or in computing the scaled segment with these new units.

Effects of differentiation in conceptual requirements are captured in results that show an overall shift down in tool use and understanding level in the reduction condition. Again, this varied by tool condition such that we see seventh graders (and fifth graders in the Graph Paper condition) using less sophisticated tool use strategies when reducing with the ruler or graph paper. While these tools worked to the seventh graders' advantage in the enlargement condition, their conceptual understandings of scale hindered their ability to use the tool as effectively for reducing. This phenomenon is mirrored in the performance of third graders when they used the replica. While the transparency of multiplicative operations with the replica is used to the third graders' advantage when enlarging, their limited scale understandings hinder their efforts in the reduction condition.

The Interplay between Shifting Forms of Mediation of Tool Use and Children's Developing Understandings of Scale Relations and the Development of Higher Psychological Processes

I posit that tool use became increasingly more sophisticated with age as children developed increasingly more sophisticated understandings in scale, and vice versa. Furthermore, I suggest that younger children were less able to mediate problem solving (i.e., determine the scaled length) with the tool, and their behavior was characteristic of "elementary processing" (Vygotsky, 1978). In other words, young children were often not able to mediate scaling with the tool, and scaling was largely determined by such biologically endowed abilities as spatial abilities. On the other hand, older children often effectively mediated problem solving with the tool functioning as a necessary external support, a *function* that Vygotsky would posit had become "interiorized" by the oldest children who computed scaled dimensions mentally. The mediated quality of these children's problem solving is characteristic of Vytgotsky's "higher psychological processing".

What is critical to note in this analysis is that shifts in tool use were directly related to shifts in understanding, and I posit that the two necessarily interplayed in the development process. I argue, for instance, that mediation of problem solving with the tool could not have shifted to "internal" status without the corresponding shifts in understandings; mental calculation of the scaled side is impossible without a multiplicative understanding of scale that can be instantiated in arithmetical--as opposed to "counting"--problem solving strategies. Likewise, effective mediation with the tool as an external support also required multiplicative understandings, however, understandings that needed only be "graphic" in nature such that scaling was a counting and measurement operation.

Applications of the Research to Educational Practice

Specific aspects of this research can inform current mathematics classroom practices. In the "reform" mathematics classroom, "teachers need to implement classroom practice built upon the ways that children interpret mathematical tasks and bring their interpretations to bear on solving mathematical problems or investigating mathematical ideas" (Saxe & Gearhart, 1995, pp. 1-2). The research reported here can help teachers understand children's difficulties in understanding scale. If teachers understand that the concepts children generate may take a variety of forms (e.g., qualitative, ordinal, or additive) before becoming multiplicative, teachers can plan activities that help students discover the limitations of their concepts, for instance, and spur them to seek new explanations.

Furthermore, the California State "Framework" (1992) and NCTM "Standards" (1991) place the importance of the use of manipulatives (tools) in the classroom to new heights. Mathematics is seen as a discipline of inquiry, characterized by "doing" mathematics; using mathematics tools to solve problems. Hence teachers should understand the ways in which mathematical tools used in scaling both constrain and enable particular types of problem solving. For instance, young children attempted more multiplicative problem solving with the replica than with the ruler or graph paper. The need to translate lengths to other units and deal with partitions and values potentially hindered younger children's efforts, whereas these same properties allowed for more complex computations and arithmetical problem solving for older, more able children.

Teachers assessments of the students' conceptual understandings would also benefit from understanding that children's tool using abilities, in addition to their understandings, potentially undergo developmental transitions. For instance, the research showed that children might have adequate understandings in scale, but were unable to organize problem solving with the tool.

Finally, the research showed that children's scale drawings, while an important indicator of their competency in scale, fell far short of telling the whole story. This is an important point since so much classroom assessment is based on the products of students problem solving rather than the process.

Conclusion

Using Leontiev's study on the mediation of memory, Vygotsky provided some insight into sign using activities. In the research reported here, I extended this analysis to the higher order domain of mathematics. I analyzed the complex ways particular tools and direction domain interacted with children's developing tool using activity and developing understandings, as well as with the effectiveness of their drawings. Finally, I analyzed the ways in which shifting forms of mediation in tool using activity and developing understandings in scale relations were interwoven in children's problem solving activity.

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Table 1

Proposed Developmental Shifts in Scale Understandings for Enlargements and Reductions

	Proposed Developmental Shifts in Understanding in enlargement & Reduction Task Domain	Pilot Work Example Strategy: To enlarge or reduce a battery dimension 4 times.	Empirical Support from the Proportion/Ratio Literature[1]
1	Scale understanding is qualitative or ordinal.	To enlarge or reduce, child draws an estimate of a battery that is qualitatively "bigger" or "smaller" but similar to the original.	I. First, in the equation A/B=C/D children may not understand the problem or only attend to part of the information such as the numerators. II. Then, children can reason about all four factors in the problem, but only in a qualitative manner. In addition, at this level children may have a visual understanding of ratio and proportion (Steffe and Parr, 1968, in Lamon, 1990), especially of congruence and similarity (Van den Brink & Streefland, 1979).
2	Scale understanding is quantitative but additive rather than multiplicative. Child knows object dimension must be larger or smaller, similar, and the scale factor must be incorporated somehow (e.g. four units added on for enlargement or subtracted for reduction)	To enlarge, child draws original dimension, then adds four "large" graph paper squares (1/4" graph paper has bold lines every 1", hence the large squares). To reduce, child subtracts four small graph paper squares from the original dimension and draws resultant length.	III. Next, children attempt to quantify results using constant additive difference strategies (e.g., A-B=C-D) versus multiplicative strategies. IV. Then children progress to what Piaget termed the "preproportionality" stage in which children still use additive strategies but not with constant differences. They realize that differences change with the size of the numbers, but they do not yet realize they need to consider a constantly increasing difference.
3	Scale understanding is quantitative and multiplicative. Child knows object dimension must be the scale factor number of times larger or smaller.	For a battery dimension 4 times larger, repeatedly adds the dimension four times or measures the dimension and multiplies the measure by 4. For a battery dimension 4 times smaller, visually partitions dimension into 4 parts or measures the dimension and divides the measure by 4.	V. Finally, children use what Piaget terms "logical proportions" in which a multiplicative relationship is noticed between two terms and this relationship is applied to the other two terms. Here children formulate a law that can be generalized to all cases.

[1] (From Inhelder & Piaget, 1958; Lesh, Post, & Behr, 1988; Tourniaire & Pulos, 1985)

Table 2

Proposed Additional Developmental Shifts in Scale-linked Understandings for Reductions

	Proposed Level of Understanding in Reduction Task Domain Only	Pilot Work Example Strategy	Empirical Support from the Fractions Literature
1	Reduction understanding is qualitative. Partitioning understandings are unsystematic; strategies reflect successive fragmentation that ignores equality of parts and remainders of parts.	For a battery dimension 4 times smaller, child draws a qualitative estimate of a battery that is smaller than but approximately similar to the original--the exact size of which is arbitrary.	I. When subdividing into parts greater than two, children seem to follow a general pattern of procedures. First they try successive fragmentation, or cutting off the required number of parts, ignoring the remainder if there is one or providing further cuts (Piaget, Inhelder, & Szeminska, 1960). The child may also attempt "algorithmic halving" without attending to equality of parts (Pothier & Sawada, 1983).
2	Reduction understanding is quantitative however dominated by "one-half" thinking. Reduction of an object to any scale is accomplished by partitioning it to half the size of the original.	For a battery dimension 4 times smaller, child draws half the length of the dimension.	II. The next strategy child uses is "successive dichotomies" (Piaget et al., 1960) or the "evenness" (Pothier & Sawada, 1983) strategy, where child successively halves object and attends to equality of parts (see also Kieren & Nelson, 1978; Kieren & Southwell, 1979).
3	Reduction understanding is quantitative; it is understood the dimension must be partitioned the scale factor number of times, and one partition equals the reduced size. Partitioning is accomplished through trial and error because the child does not have an effective operational anticipatory schema for partitioning.	For a battery dimension 4 times smaller, child visually divides units of the dimension into four parts. For example, a child using graph paper squares partitions squares in a trial and error manner (maybe dividing by units of 1 square first, then 2 squares, etc.) until 4 equal parts are obtained. He/she then draws the reduced dimension equal to one of the parts.	III. First "cut" is not longer dominated by "one-half" thinking, but partitions are obtained in a trial and error manner(Piaget et al., 1960). For Pothier and Sawada (1983) this is the "oddness" strategy.

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4	Reduction understanding is quantitative as in 3, however child has an effective operational anticipatory schema for partitioning. This partitioning reflects understanding that fractions imply a nesting system; they are parts of the whole and parts themselves that can be further subdivided. Where quantities are available for partitioning, division algorithms are used.	For drawing a battery dimension 4 times smaller, child measures dimension and divides the numerical calculation by 4. He/she then draws the calculated length of the dimension. If no numerals are available (e.g. when using only the continuous replica of the battery) the battery dimension can be folded once, then folded again to accomplish reduction (this was not observed in pilot work, but may be observed in the proposed study since a "replica only" condition will exist).	IV. Children have an operational anticipatory schema for partitioning guided by coordination of all seven characteristics of a fraction (see Piaget et al., 1960 for characteristics). Pothier & Sawada (1983) call this the "composition" strategy or "multiplicative algorithm" where, for example for 1/9, children divide the object into thirds then trisect each third.

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Table 3

Tool Use Coding Scheme

1	The child is either unable to organize activity with the tool to mediate an effective solution (e.g., child draws some arbitrary larger or smaller F without using the tool) or the tool actually hinders problem solving activity.
2	The child organizes the tool to mediate an ineffective solution, and the tool is an external support. (e.g., the child adds 4 inches to a side when using a ruler). The tool serves an instrumental function in problem solving.
3	Children are able to use the tool to mediate problem solving in effective ways, however the tool is largely an external support (e.g., the child moves the ruler iteratively 4 times). Tool is necessary for initial and final measurement of side as well as to the calculation of scaled length.
4	Tool using activity progressed to where problem solving is mediated in part internally. (e.g., child measures side, multiplies by 4 and draws). Scaled length is computed mentally so tool is only necessary for initial and final measurement of side.

Table 4

Enlargement Understanding Coding Scheme

0	Child draws scaled side smaller than or equal to the original side. Does not understand scale. *Note: for analysis, 0 & 1 are collapsed and = 1*
1	Scale understanding is qualitative or ordinal, i.e., child knows scaled object must be larger, and similar, but size is arbitrary (e.g., draws an estimate of an F that is qualitatively "bigger" but similar to the original, or draws the original side and then draws a bit more, adding onto it)
2	Scale understanding is quantitative but additive rather than multiplicative. Child knows object dimension must be larger, similar, and the scale factor must be incorporated somehow. Generally an additive strategy involves the child adding 4 units to the original length (e.g. draws original dimension, then adds four "large" graph paper squares).
3	Scale understanding is quantitative but somewhere between additive and multiplicative. Child draws one side and adds 4 more, enlarging the side 5 times.
4	Scale understanding is quantitative and multiplicative. Child knows object dimension must be the scale factor number of times larger (e.g., repeatedly adds the dimension four times or measures the dimension and multiplies the measure by 4).

Table 5

Reduction Understanding Coding Scheme

0	Child draws scaled side larger than or equal to the original side. Does not understand scale. *Note: for analysis, 0 & 1 are collapsed and = 1*
1	Reduction understanding is qualitative. Partitioning understandings are unsystematic; strategies reflect successive fragmentation that ignores equality of parts and remainders of parts (e.g., draws a qualitative estimate of an F that is smaller than but approximately similar to the original--the exact size of which is arbitrary.)
2	Reduction understanding is quantitative however dominated by "one-half" thinking. Reduction of an object to any scale is accomplished by partitioning it to half the size of the original. (e.g., draws half the length of the dimension).
3	Reduction understanding is quantitative and additive. Reduction of the object is accomplished by, for instance, subtracting 4 units from the measurement of the side, or subtracting 3 units so there is 1 left, or subtracting 4 partitions so there is zero left.
4	Reduction understanding is quantitative and multiplicative; it is understood the dimension must be partitioned the scale factor number of times, and one partition equals the reduced size. Partitioning is accomplished through trial and error because the child does not have an effective operational anticipatory schema for partitioning. (e.g., child visually divides units of the dimension into four parts. For example, a child using graph paper squares partitions squares in a trial and error manner--maybe dividing by units of 1 square first, then 2 squares, etc.--until 4 equal parts are obtained. He/she then draws the reduced dimension equal to one of the parts). Not evident that child knows the numeric length of the segment until drawing is completed.
5	Reduction understanding is quantitative and multiplicative as in 4, however child has an effective operational anticipatory schema for partitioning. This partitioning reflects understanding that fractions imply a nesting system; they are parts of the whole and parts themselves that can be further subdivided. Where quantities are available for partitioning, division algorithms may be used. (e.g., child measures dimension and divides the numerical calculation by 4. He/she then draws the calculated length of the dimension). Child knows the length of the segment. Or, the child may simply divide the object in 1/2 and then in 1/2 again.

Figure 1

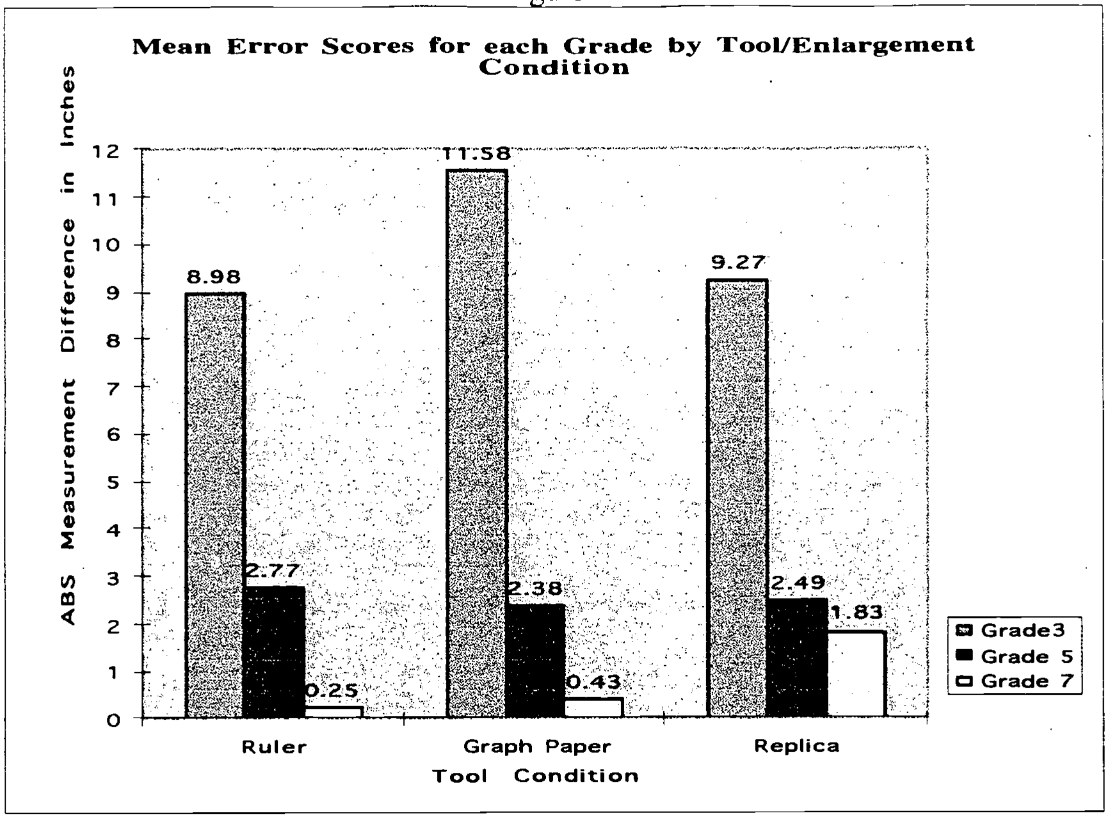
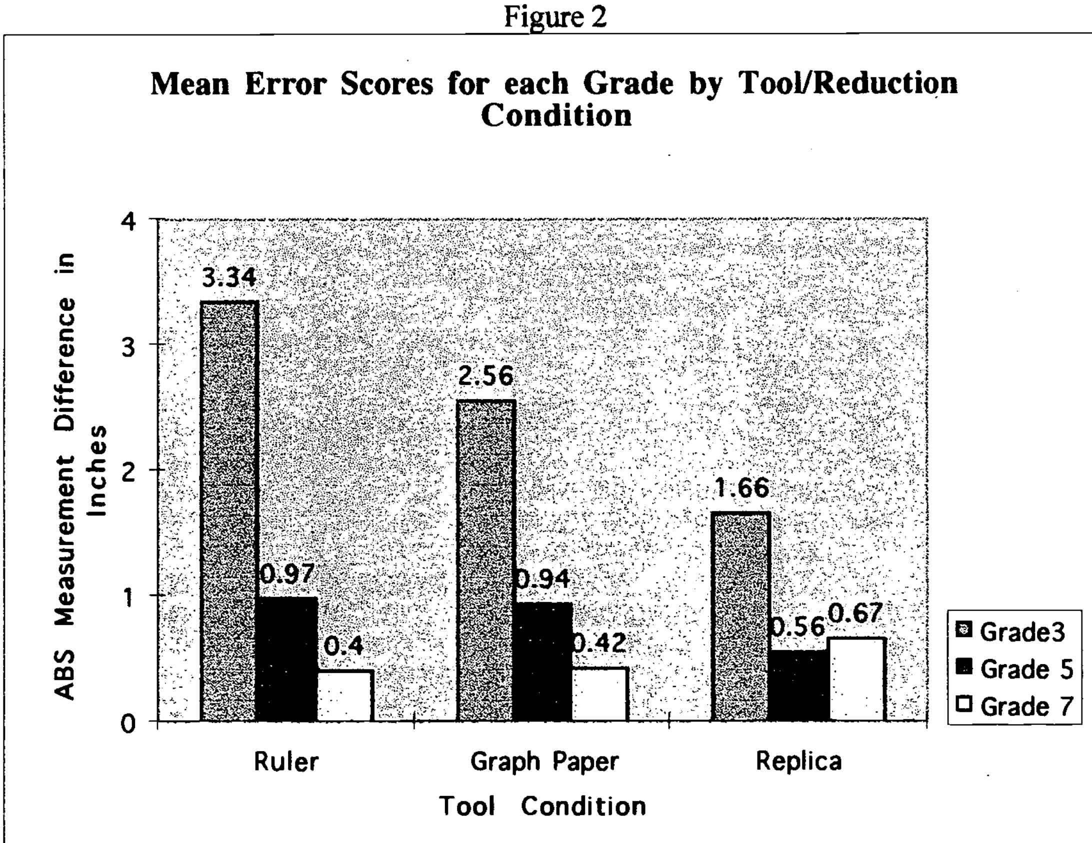
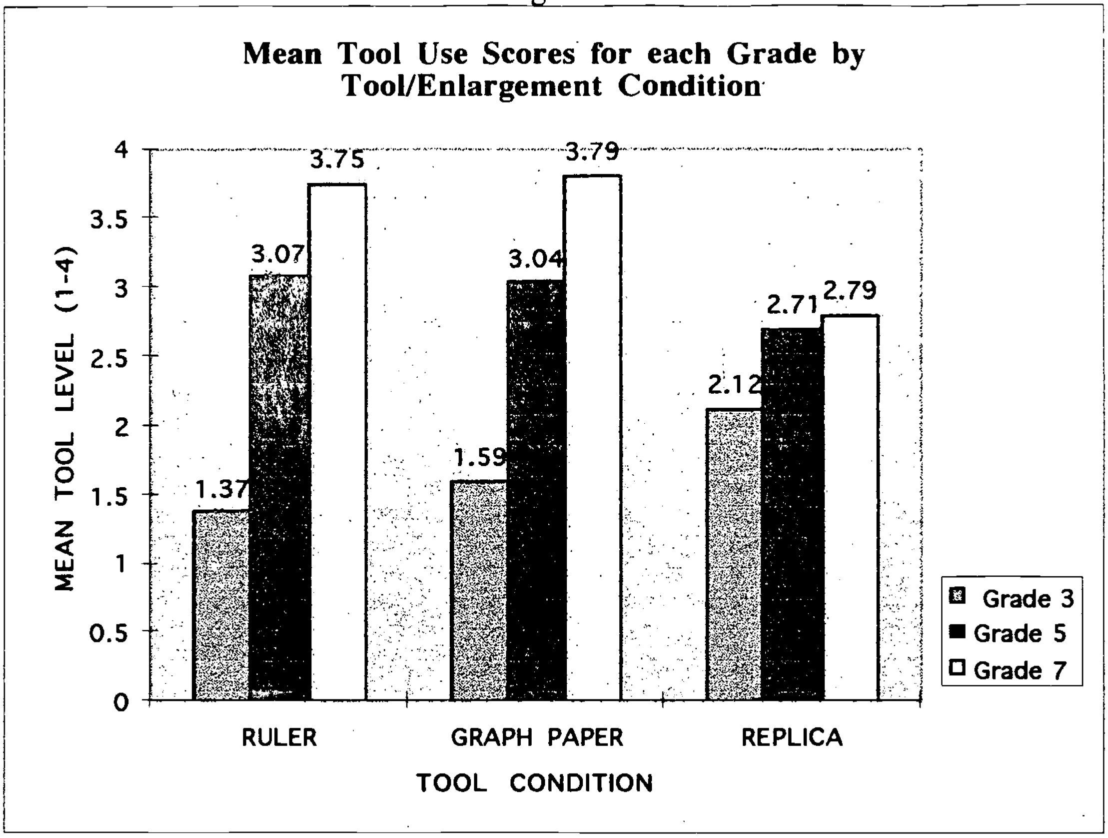


Figure 2



Grade 3 n=19, Grade 5 n=18, Grade 7 n=17.
Note: one outlier for the 7th grade was eliminated from these analyses.
Note: The higher the Error score, the less accurate the drawing.
See Appendix for Standard Deviations

Figure 3



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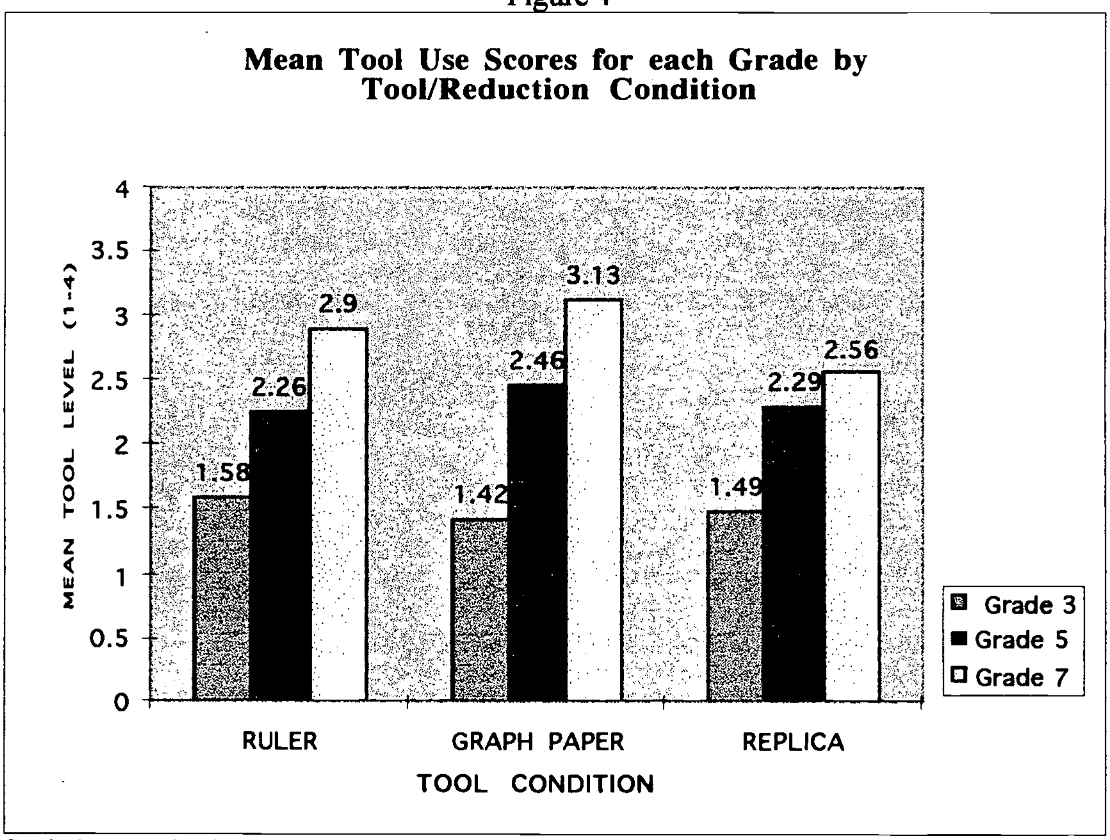
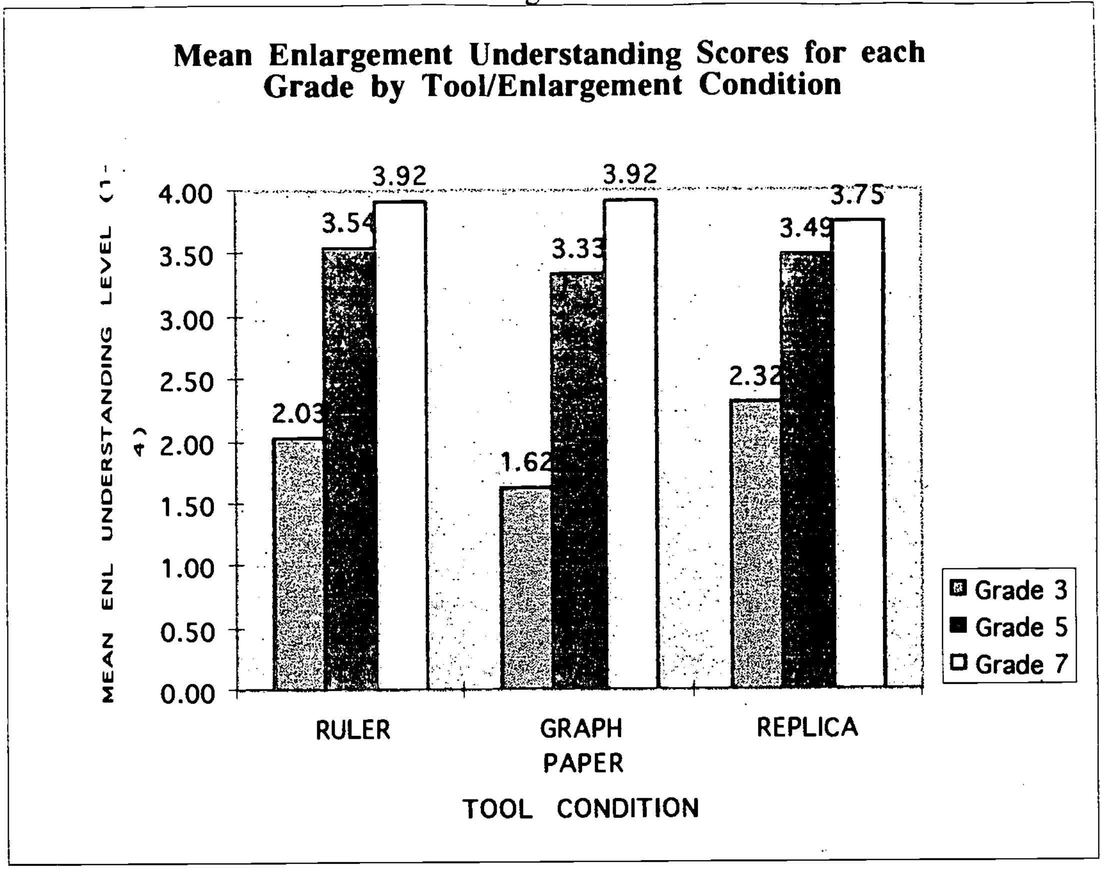


Grade 3 n=19, Grade 5 n=18, Grade 7 n=18.
See Appendix G for Standard Deviations

Figure 5



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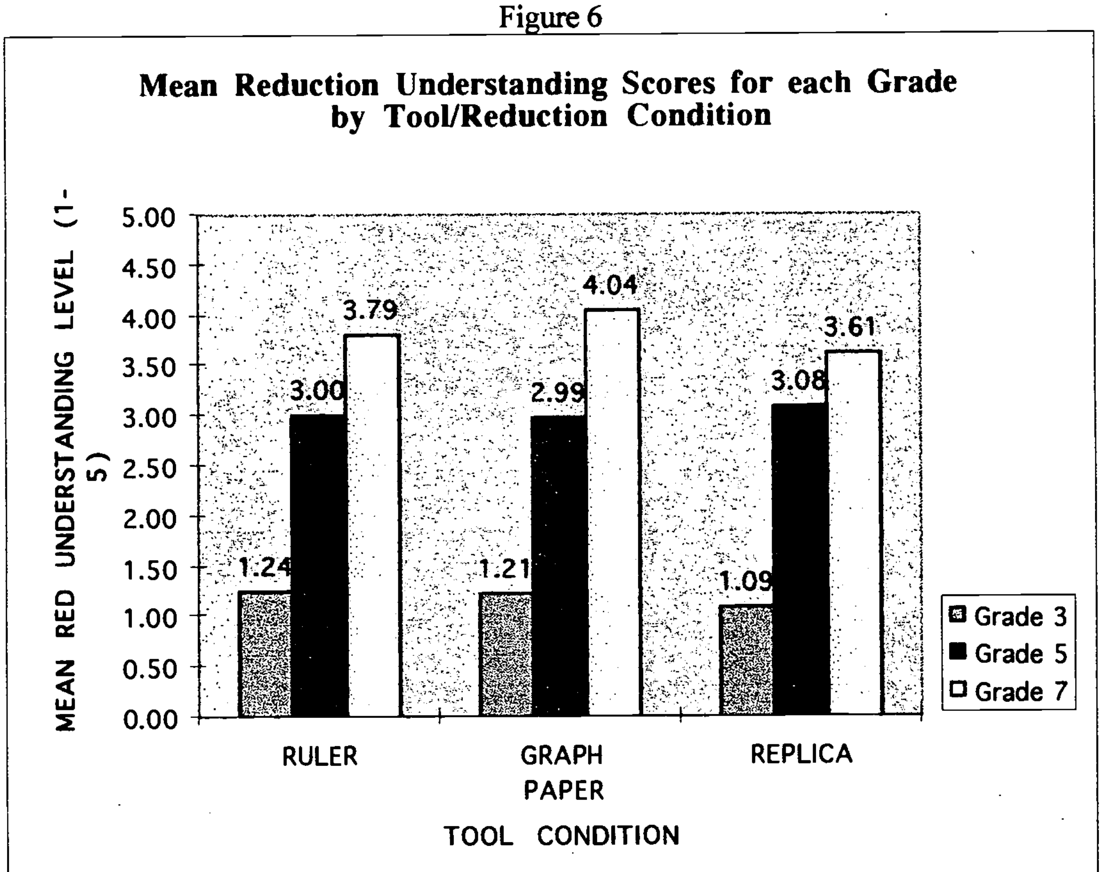


Grade 3 n=19, Grade 5 n=18, Grade 7 n=18.
See Appendix G for Standard Deviations

Figure 7

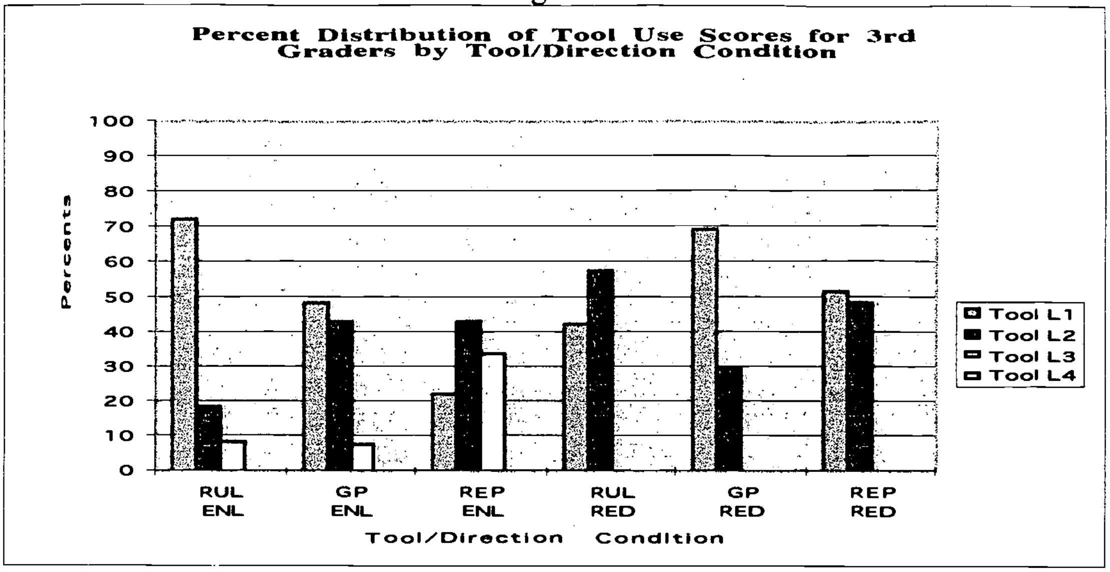


Figure 8

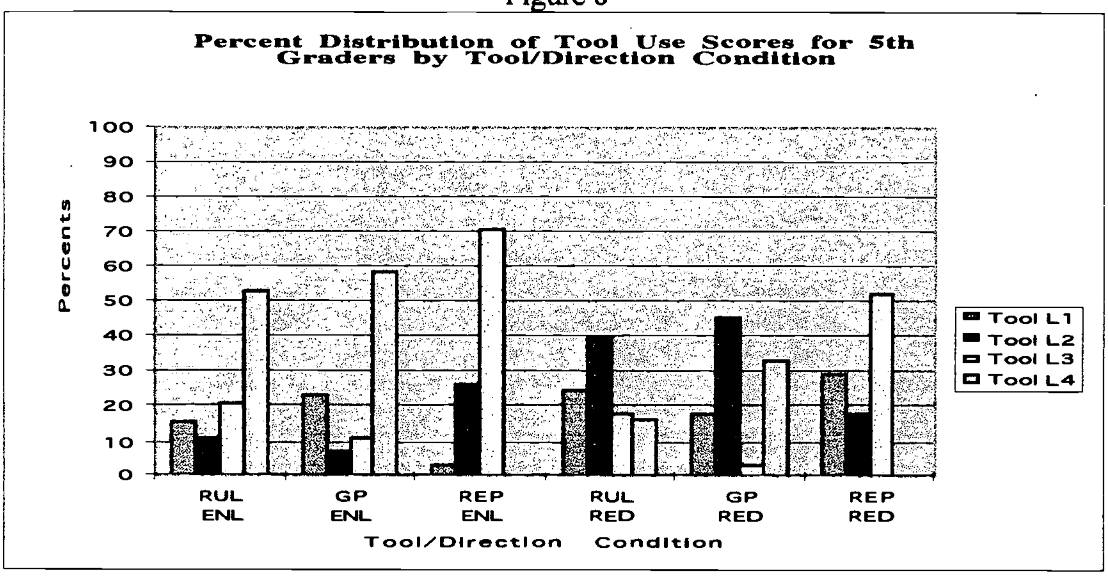
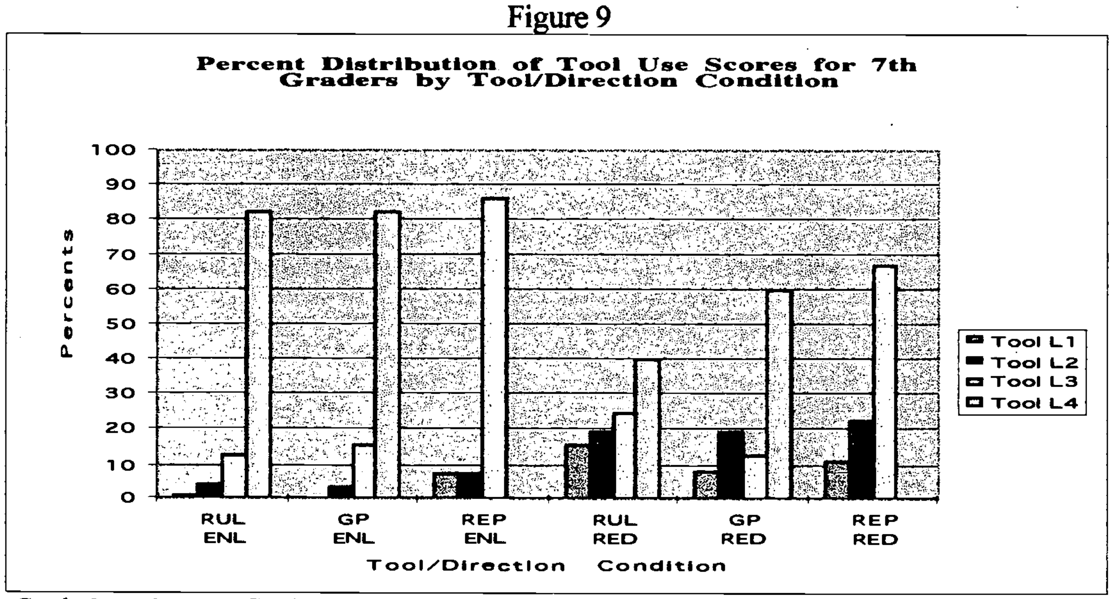


Figure 9



Grade 3 n=19 Grade 5 n=18 Grade 7 n=18

Rul=ruler, GP=graph paper, Rep=replica ---- Enl=enlargement, Red=reduction

Figure 10

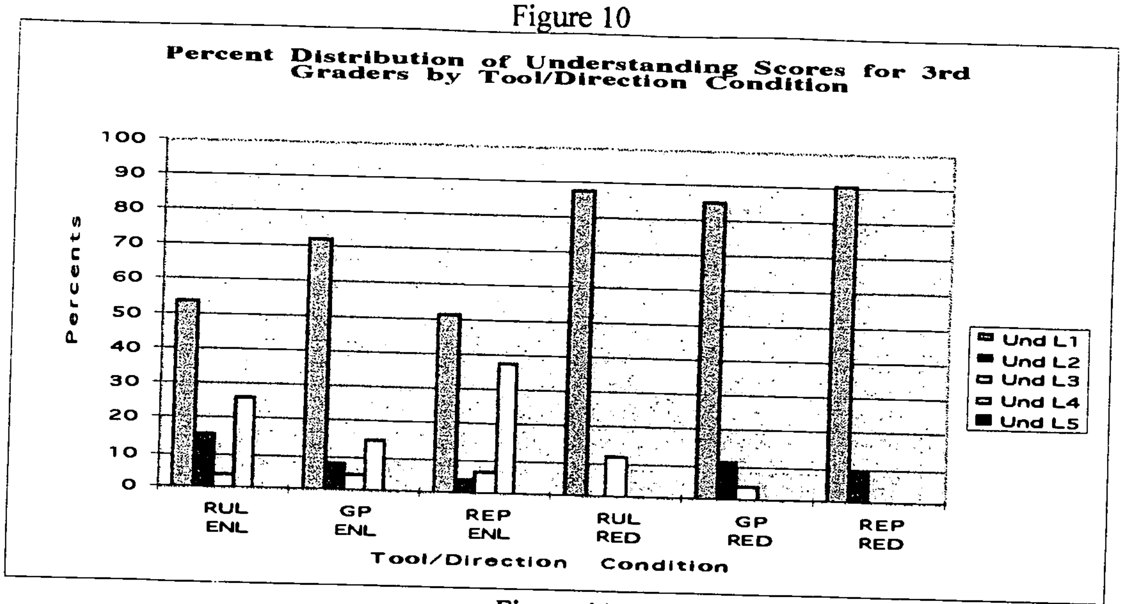


Figure 11

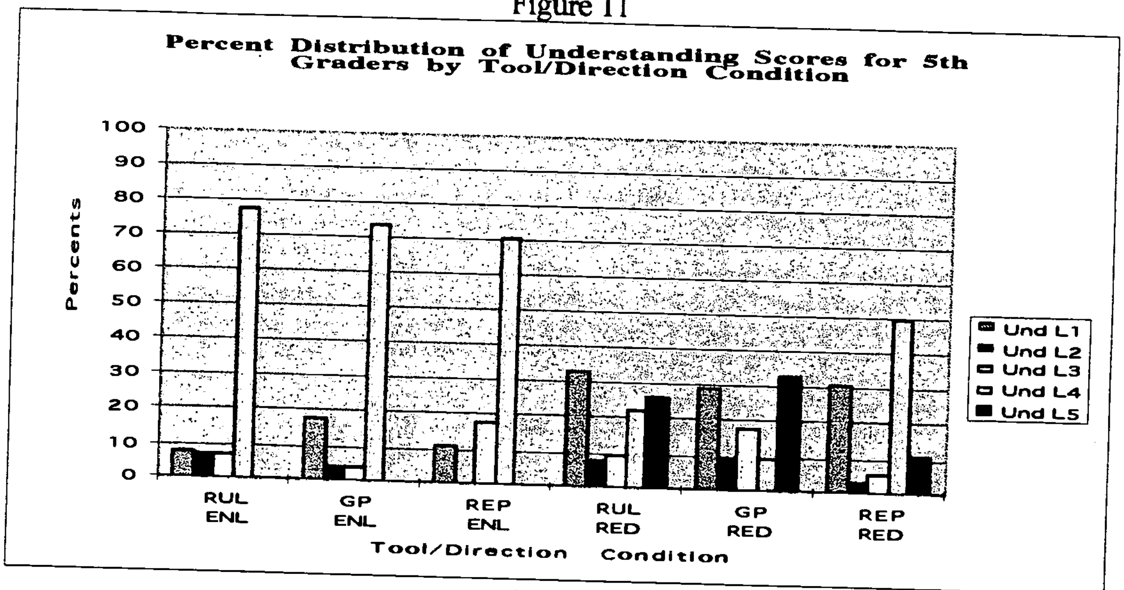
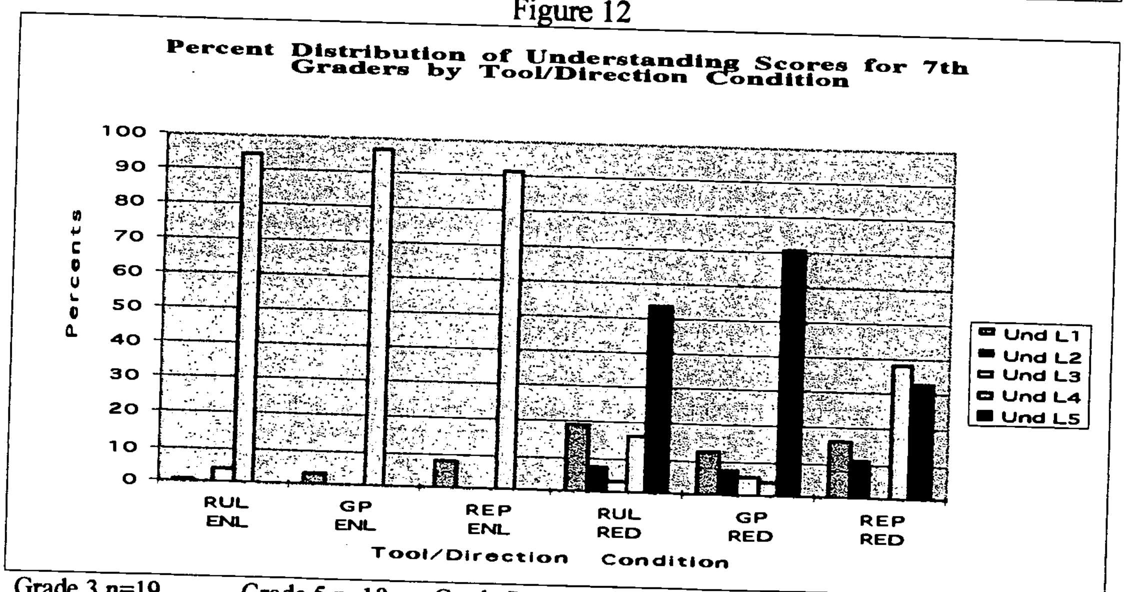


Figure 12



Grade 3 n=19 Grade 5 n=18 Grade 7 n=18
Rul=ruler, GP=graph paper, Rep=replica ---- Enl=enlargement, Red=reduction

Appendix

Means and Standard Deviations of all Dependent Variables

Error Score Means and Standard Deviations by Grade and Tool/Direction Condition

	ENLARGEMENTS						REDUCTIONS					
	Ruler		Graph Paper		Replica		Ruler		Graph Paper		Replica	
Grade	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
3 n=19	8.98	4.21	11.58	4.51	9.27	6.62	3.34	2.58	2.56	1.86	1.66	1.05
5 n=18	2.77	3.10	2.38	3.16	2.49	1.71	.97	.97	.94	.89	.56	.65
7 n=17	.25	.66	.43	.85	1.83	2.50	.40	.51	.42	.61	.67	1.34

Note: for grade 7 one outlier was eliminated from analyses.

Tool Use Score Means and Standard Deviations by Grade and Tool/Direction Condition

	ENLARGEMENTS						REDUCTIONS					
	Ruler		Graph Paper		Replica		Ruler		Graph Paper		Replica	
Grade	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
3 n=19	1.37	.47	1.59	.51	2.12	.53	1.58	.37	1.42	.34	1.49	.42
5 n=18	3.07	.95	3.04	1.14	2.7	.46	2.26	.97	2.46	.92	2.29	.91
7 n=18	3.75	.56	3.79	.46	2.79	.51	2.9	.99	3.13	.92	2.56	.64

Understanding Score Means and Standard Deviations by Grade and Tool/Direction Condition

	ENLARGEMENTS						REDUCTIONS					
	Ruler		Graph Paper		Replica		Ruler		Graph Paper		Replica	
Grade	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
3 n=19	2.03	.96	1.62	.85	2.32	1.09	1.24	.63	1.21	.44	1.09	.25
5 n=18	3.54	.67	3.33	.96	3.49	.59	3.00	1.54	2.99	1.42	3.08	1.38
7 n=18	3.92	.35	3.92	.35	3.75	.58	3.79	1.61	4.04	1.44	3.61	1.43

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References

Behr, M. J., Lesh, R., Post, T. R., & Silver, E. A. (1983). Rational-number concepts. In R. Lesh & M. Landau (Eds.), Acquisition of mathematics concepts and processes (pp. 92-127). New York: Academic Press.

Behr, M. J., Harel, & Post, T. R. (1992). Rational number, ratio, and proportion. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics (pp. 296-333). New York: Macmillan Publishing.

California Department of Education. (1992). Mathematics framework for California public schools. Sacramento, CA: Author.

Carpenter, T. P., Fennema, E., & Romberg, T. A. (Eds.). (1993). Rational numbers: An integration of research. Hillsdale, NJ: Lawrence Erlbaum Associates.

Carraher, T. N. (1986). From drawings to buildings; working with mathematical scales. International Journal of Behavioral Development, 9, 527-544.

Cobb, P. (1993, April). Cultural tools and mathematical learning: A case study. Paper presented at the Annual Meeting of the American Educational Research Association, Atlanta, GA.

Cobb, P. (1995). Where is the mind? Constructivist and sociocultural perspectives on mathematical development, Educational Researcher, 23 (7), 13-20.

Cole, M., & Cole, S. (1989). The development of children. New York: Scientific American Books.

Cole, M. & Griffin, P. (1980). Cultural amplifiers reconsidered. In D. Olson (Ed.), Social foundations of language and thought. New York: Norton.

DeLoache, J, (1991a) Physical similarity and young children's understanding of scale models. Child Development, 62(1), 111-126.

DeLoache, J, (1991b). Symbolic functioning in very young children: Understanding of pictures and models. Child Development, 62(4), 736-762.

Ginsburg, H. P., & Opper, S. (1988). Piaget's theory of intellectual development: An introduction. Third Edition. New Jersey: Prentice-Hall.

Goldenberg, E. P. (1988a). Mathematics, metaphors, and human factors: Mathematical, technical, and pedagogical challenges in the educational use of graphical representation of functions. Journal of Mathematical Behavior, 7, 135-173.

Goldenberg, E. P. (1988b). Metaphors for understanding graphs: What you see is what you see. (Report No. ETC-TR-88-22). Washington, DC: Office of Educational Research and Improvement. (ERIC Document Reproduction Service No. ED 303 369)

Greenfield, P. M. (1995). Cultural tools and learning processes in a changing world. Final report to the Spencer Foundation.

Guberman, S. R. (1992). Math and money: A comparative study of the arithmetical achievements and out of school activities of Latino and Korean American children. Unpublished doctoral dissertation, University of California, Los Angeles.

Guershon, H., & Confrey, J. (Eds.). (1994). The development of multiplicative reasoning in the learning of mathematics. New York: State University of New York.

Hart, K. M. (1984). Ratio: Children's strategies and errors. A report of the strategies and errors in secondary mathematics project. Windsor, Berkshire: Nfer-Nelson.

Hart, K. M. (1988). Ratio and proportion. In M. Behr & J. Hiebert (Eds.), Number concepts in the middle grades (pp. 198-219). National Council of Teachers of Mathematics: Lawrence Erlbaum.

Inhelder, B. & Piaget, J. (1958). The growth of logical thinking from childhood to adolescence. New York: Basic Books.

Karplus, R., Pulos, S., & Stage, E. K. (1983). Proportional reasoning of early adolescents. In R. Lesh & M. Landau (Eds.), Acquisition of mathematics concepts and processes (pp. 45-90). New York: Academic Press.

Kieren, T., & Nelson, D. (1978). The operator construct of rational numbers in children and adolescents. Alberta Journal of Educational Psychology, 24, 22-30.

Kieren, T., & Southwell, B. (1979). The development in children and adolescents of the construct of rational numbers as operators. Alberta Journal of Educational Psychology, 25, 234-247.

Lamon, S. J. (1990). Ratio and proportion: Cognitive foundations in unitizing and norming. Paper presented at the Annual Meeting of the American Educational Research Association, Boston, MA. (ERIC Document Reproduction Service No. ED 325 335)

Lamon, S. J. (1993). Ratio and proportion: Connecting content and children's thinking. Journal for Research in Mathematics Education, 24(1), 41-61.

Lamon, S. J. (1994). Ratio and proportion: Cognitive foundations in unitizing and norming. In H. Guershon & J. Confrey (Eds.), The development of multiplicative reasoning in the learning of mathematics (pp. 89-120). Albany, NY: State University of New York Press.

Lave, J. (1988). Cognition in practice: Mind, mathematics and culture in everyday life. Cambridge University Press.

Lave, J., Murtaugh, M., & de la Rocha, O. (1984). The dialectic of arithmetic in grocery shopping. In B. Rogoff & J. Lave (Eds.), Everyday cognition: Its development in social context (pp. 151-171). Cambridge, MA: Harvard University Press.

Lesh, R., Post, T., & Behr, M. (1988). Proportional Reasoning. In M. Behr & J. Hiebert (Eds.), Number concepts in the middle grades (pp. 93-118). National Council of Teachers of Mathematics: Lawrence Erlbaum.

Millroy, W. (1992). An ethnographic study of the mathematical ideas of a group of carpenters. Journal for Research in Mathematics, Monograph number 5. Reston, VA: National Council of Teachers of Mathematics.

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National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston, VA: Author.

Noelting, G. (1980a). The development of proportional reasoning and the ratio concept: Part I: Differentiation of stages. Educational Studies in Mathematics, 11, 217-253.

Noelting, G. (1980b). The development of proportional reasoning and the ratio concept. Part II: Problem solving structure at successive stages; problem-solving strategies and the mechanism of adaptive restructuring. Educational Studies in Mathematics, 11, 331-363.

Oxford English Dictionary (1989). Prepared by J. A. Simpson and E. S. C. Weiner. Oxford: Clarenden Press.

Pea, R. D. (1993). Practices of distributed intelligence and designs for education. In G. Salomon (Ed.), Distributed cognitions: Psychological and educational considerations (pp. 47-87). New York: Cambridge University Press.

Piaget, J., Inhelder, B., & Szeminska, A. (1960). The child's conception of geometry. New York: Basic Books.

Pothier, Y., & Sawada, D. (1983). Partitioning: The emergence of rational number ideas in young children. Journal for Research in Mathematics Education, 14(4), 307-317.

Rogoff, B. (1990). Apprenticeship in thinking. New York: Oxford University Press.

Rogoff, B., & Gardner, W. (1984). Adult guidance of cognitive development. In B. Rogoff & J. Lave (Eds.), Everyday cognition: Its development in social context (pp. 151-171). Cambridge, MA: Harvard University Press.

Rogoff, B., & Lave, J. (Eds.). (1984). Everyday cognition. Cambridge, MA: Harvard University Press.

Saxe, G. B. (1991). Culture and cognitive development: Studies in mathematical understandings. Hillsdale, NJ: Lawrence Erlbaum.

Saxe, G. B., & Gearhart, M. (1995). Teacher's practices and children's mathematics: A focus on measurement and scale. Research proposal for the James S. McDonnel Foundation.

Saxe, G. B., & Moylan, T. (1982). The development of measurement operations among the Oksapmin of Papua New Guinea. Child Development, 53, 1242-1248.

Sloan, T.F. (1996). An analysis of developmental shifts in children's tool use and conceptual understandings in scale drawing. Unpublished doctoral dissertation, University of California, Los Angeles.

Smagorinski, P. (1995). The social construction of data: Methodological problems of investigating learning in the zone of proximal development. Review of Educational Research, 65 (3), 191-212.

Tourniaire, F., & Pulos, S. (1985). Proportional reasoning: A review of the literature. Educational Studies in Mathematics, 16, 181-204.

Van Den Brink, J., & Streefland, L. (1979). Young children (6-8): Ratio and proportion. Educational Studies in Mathematics, 10, 403-420.

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Eds. M. Cole, V. John-Steiner, S. Scribner, & E. Souberman. Cambridge, MA: Harvard University Press.

Vygotsky, L. S. (1986). Thought and language. Cambridge MA: The MIT Press.

Wertsch, J. (1991). Voices of the mind: A sociocultural approach to mediated action. Cambridge, MA: Harvard University Press.

Wertsch, J., Minick, N., & Arns, F. J. (1984). The creation of context in joint problem solving. In B. Rogoff & J. Lave (Eds.), Everyday cognition: Its development in social context (pp. 151-171). Cambridge, MA: Harvard University Press.

Wertsch, J. (1985). Vygotsky and the social formation of mind. Cambridge, MA: Harvard University Press.

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